PHY420 Problems Class 1: Random Walks & Diffusion

Dr. Rhoda Hawkins

1. Consider the diffusion equation, which is the partial differential equation:

$$\frac{\partial c(x,t)}{\partial t} = D\frac{\partial^2 c(x,t)}{\partial x^2}.$$

   Test, by direct substitution, that

$$c(x,t) = \frac{1}{\sqrt{4\pi Dt}}\exp\left[-\frac{(x-x_0)^2}{4Dt}\right]$$

   is a solution, assuming that $t > 0$.

2. It can be shown that the probability that a one dimensional random walk, starting at site 0, has reached site n after N steps is given by the binomial distribution:

$$P(n;N) = \frac{1}{2^N}\left(\begin{array}{c} N \\ \frac{1}{2}(N+n) \end{array}\right) = \frac{N!}{(\frac{1}{2}(N+n))!(n-\frac{1}{2}(N+n))!}.$$

   Use *Stirling's approximation*, $\ln N! = N\ln N - N$ (on formula sheet) to show that this reduces to a *Gaussian* when $N \gg 1$ and $n/N \ll 1$.

3. Gaussian integrals feature in this course. It is useful to be able to calculate them easily.

   (a) The Gaussian integral is (as given on your formula sheet):

$$I = \int_{-\infty}^{+\infty} \mathrm{d}x\; e^{-x^2} = \sqrt{\pi}.$$

   Prove this by writing down I^2 and transforming to *polar coordinates*. You should now be able to integrate explicitly.

(b) Calculate

$$I(a) = \int_{-\infty}^{+\infty} \mathrm{d}x \;\; e^{-\frac{a}{2}x^2}$$

by transforming the integration variable and using the result given in (a).

(c) Use the trick of "completing the square" to calculate:

$$I(a,b) = \int_{-\infty}^{+\infty} \mathrm{d}x \;\; e^{-\frac{a}{2}x^2 - bx}.$$

(d) Finally, use the knowledge you have gained above to compute:

$$K = \int_{-\infty}^{+\infty} \mathrm{d}x_1 \;\; \exp\left[-\frac{a}{2}(x_0 - x_1)^2 - \frac{b}{2}(x_1 - x_2)^2\right].$$