PHY420 Problems Class 1: Random Walks & Diffusion

Dr. Rhoda Hawkins

1. Consider the diffusion equation, which is the partial differential equation:

$$\frac{\partial c(x,t)}{\partial t} = D\frac{\partial^2 c(x,t)}{\partial x^2}.$$

Test, by direct substitution, that

$$c(x,t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left[-\frac{(x-x_0)^2}{4Dt}\right]$$

is a solution, assuming that $t > 0$.

2. It can be shown that the probability that a one dimensional random walk, starting at site 0, has reached site n after N steps is given by the binomial distribution:

$$P(n) = \frac{1}{2^N}\frac{N!}{(\frac{1}{2}(N+n))!(\frac{1}{2}(N-n))!}.$$

Use *Stirling's approximation*, $\ln N! = N\ln N - N$ (on formula sheet) to show that this reduces to a *Gaussian* when $N \gg 1$ and $n/N \ll 1$.
You may use $\ln(1+x) = x - x^2/2 + O(x^3)$ for small x.
Note that $n = x/a$ where x is the position at site n and a is the step size.