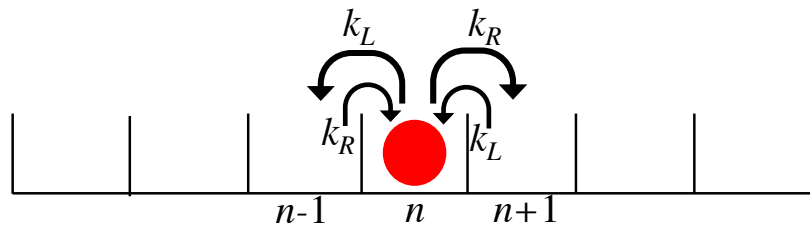


PHY420 Problems Class 5: Molecular Motors

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1. A motor is modeled as a point particle which jumps from site to site on a one-dimensional lattice. The motor has probability per unit time (rate) k_R to jump to the next site on the right and k_L to jump to the next site on the left. The distance between two sites is a . We call $P_n(t)$ the probability that the motor is at site n at time t .



- (a) Show that the Master equation for $P_n(t)$ is

$$\frac{\partial P_n}{\partial t} = k_R(P_{n-1} - P_n) + k_L(P_{n+1} - P_n).$$

[2]

- (b) Take the continuum limit of the equation in part (a) by defining a spatial coordinate $z = na$ so $P_{n+1} \rightarrow P(z+a)$ where a is the step size (NB this is not the same as a in the formula sheet form of the Taylor series). Then by doing Taylor expansions up to second order in $a \ll z$ show that you get the following Fokker-Planck equation of the form

$$\frac{\partial P(z, t)}{\partial t} = -v \frac{\partial P(z, t)}{\partial z} + D \frac{\partial^2 P(z, t)}{\partial z^2}$$

where the average velocity, v , and the diffusion constant, D , are constants to be found.

[3]

- (c) If the motor is subject to an external force f , the rates k_R and k_L change with force as $k_R = k_R^0 \exp(\frac{fc}{k_B T})$ and $k_L = k_L^0 \exp(-\frac{fb}{k_B T})$, where c and b are two microscopic lengths (of order a). We first consider the case where $k_R^0 = k_L^0$. Is this really a motor? Why?

[1]

- (d) For $k_R^0 = k_L^0$, calculate the friction coefficient relating the force to the velocity at small force to leading order in f , without assuming equilibrium. [3]
- (e) For the case of small force considered in (d), when is the Einstein relation satisfied? Can you give a simple explanation of the result? [3]
- (f) For the small force case considered in the previous parts, what is the friction constant if $k_R^0 \neq k_L^0$? Is the Einstein relation satisfied in this case? Why? [3]
- (g) What is the stall force where the average velocity of the motor vanishes? [2]