PHY420 Problems Class 5: Molecular Motors

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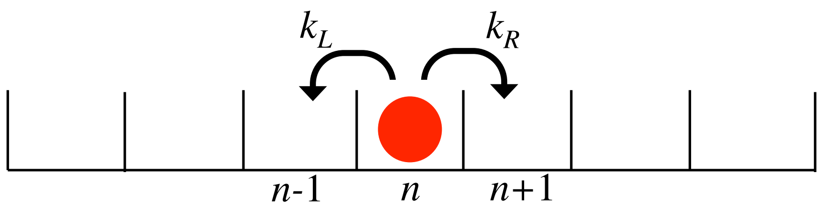
1. A motor is modeled as a point particle which jumps from site to site on a one-dimensional lattice. The motor has probability per unit time k_R to jump to the next site on the right and k_L to jump to the next site on the left. The distance between two sites is a. We call $P_n(t)$ the probability that the motor is at site n at time t.



   (a) Show that the Master equation for $P_n(t)$ is

   $$\frac{\partial P_n}{\partial t} = k_R(P_{n-1} - P_n) + k_L(P_{n+1} - P_n).$$

   [2]

   (b) Take the continuum limit for n by doing Taylor expansions in n up to second order and defining a spatial coordinate $z = na$. Show that you get the following Fokker-Planck equation of the form

   $$\frac{\partial P(z,t)}{\partial t} = -v\frac{\partial P(z,t)}{\partial z} + D\frac{\partial^2 P(z,t)}{\partial z^2}$$

   where the average velocity, v, and the diffusion constant, D, are constants to be found. [3]

   (c) If the motor is subject to an external force f, the rates k_R and k_L change with force as $k_R = k_R^0 \exp\left(\frac{fc}{k_BT}\right)$ and $k_L = k_L^0 \exp\left(-\frac{fb}{k_BT}\right)$, where c and b are two microscopic lengths (of order a). We first consider the case where $k_R^0 = k_L^0$. Is this really a motor? Why? [1]

(d) For $k_R^0 = k_L^0$, calculate the friction coefficient relating the force to the velocity at small force. [3]

(e) When is the Einstein relation satisfied? Can you give a simple explanation of the result? [3]

(f) What is the friction constant if $k_R^0 \neq k_L^0$? Is the Einstein relation satisfied in this case? Why? [3]

(g) What is the stall force where the average velocity of the motor vanishes? [2]