PHYSICAL CONSTANTS & MATHEMATICAL FORMULAE

Physical Constants

electron charge	$e = 1.60\times10^{-19}$ C
electron mass	$m_e = 9.11\times10^{-31}$ kg $= 0.511$ MeV c^{-2}
proton mass	$m_p = 1.673\times10^{-27}$ kg $= 938.3$ MeV c^{-2}
neutron mass	$m_n = 1.675\times10^{-27}$ kg $= 939.6$ MeV c^{-2}
Planck's constant	$h = 6.63\times10^{-34}$ J s
Dirac's constant ($\hbar = h/2\pi$)	$\hbar = 1.05\times10^{-34}$ J s
Boltzmann's constant	$k_B = 1.38\times10^{-23}$ J K^{-1} $= 8.62\times10^{-5}$ eV K^{-1}
speed of light in free space	$c = 299\,792\,458$ m s^{-1} $\approx 3.00\times10^{8}$ m s^{-1}
permittivity of free space	$\varepsilon_0 = 8.85\times10^{-12}$ F m^{-1}
permeability of free space	$\mu_0 = 4\pi\times10^{-7}$ H m^{-1}
Avogadro's constant	$N_A = 6.02\times10^{23}$ mol^{-1}
gas constant	$R = 8.32$ J mol^{-1} K^{-1}
ideal gas volume (STP)	$V_0 = 22.4$ l mol^{-1}
gravitational constant	$G = 6.67\times10^{-11}$ N m^2 kg^{-2}
Rydberg constant	$R_\infty = 1.10\times10^{7}$ m^{-1}
Rydberg energy of hydrogen	$R_H = 13.6$ eV
Bohr radius	$a_0 = 0.529\times10^{-10}$ m
Bohr magneton	$\mu_B = 9.27\times10^{-24}$ J T^{-1}
fine structure constant	$\alpha \approx 1/137$
Wien displacement law constant	$b = 2.898\times10^{-3}$ m K
Stefan's constant	$\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}
radiation density constant	$a = 7.55\times10^{-16}$ J m^{-3} K^{-4}
mass of the Sun	$M_\odot = 1.99\times10^{30}$ kg
radius of the Sun	$R_\odot = 6.96\times10^{8}$ m
luminosity of the Sun	$L_\odot = 3.85\times10^{26}$ W
mass of the Earth	$M_\oplus = 6.0\times10^{24}$ kg
radius of the Earth	$R_\oplus = 6.4\times10^{6}$ m

Conversion Factors

1 u (atomic mass unit) $= 1.66\times10^{-27}$ kg $= 931.5$ MeV c^{-2}	1 Å (angstrom) $= 10^{-10}$ m
1 astronomical unit $= 1.50\times10^{11}$ m	1 g (gravity) $= 9.81$ m s^{-2}
1 eV $= 1.60\times10^{-19}$ J	1 parsec $= 3.08\times10^{16}$ m
1 atmosphere $= 1.01\times10^{5}$ Pa	1 year $= 3.16\times10^{7}$ s

Polar Coordinates

$$x = r\cos\theta \qquad y = r\sin\theta \qquad \mathrm{d}A = r\ \mathrm{d}r\,\mathrm{d}\theta$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

Spherical Coordinates

$$x = r\sin\theta\cos\phi \qquad y = r\sin\theta\sin\phi \qquad z = r\cos\theta \qquad \mathrm{d}V = r^2\sin\theta\ \mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

Calculus

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
x^n	nx^{n-1}	$\tan x$	$\sec^2 x$
e^x	e^x	$\sin^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2-x^2}}$
$\ln x = \log_e x$	$\frac{1}{x}$	$\cos^{-1}\left(\frac{x}{a}\right)$	$-\frac{1}{\sqrt{a^2-x^2}}$
$\sin x$	$\cos x$	$\tan^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2+x^2}$
$\cos x$	$-\sin x$	$\sinh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{x^2+a^2}}$
$\cosh x$	$\sinh x$	$\cosh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{x^2-a^2}}$
$\sinh x$	$\cosh x$	$\tanh^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2-x^2}$
$\text{cosec}\, x$	$-\text{cosec}\, x\ \cot x$	uv	$u'v + uv'$
$\sec x$	$\sec x\tan x$	u/v	$\frac{u'v-uv'}{v^2}$

Definite Integrals

$$\int_0^\infty x^n e^{-ax}\,\mathrm{d}x = \frac{n!}{a^{n+1}} \qquad (n \geq 0 \text{ and } a > 0)$$

$$\int_{-\infty}^{+\infty} e^{-ax^2}\,\mathrm{d}x = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{+\infty} x^2 e^{-ax^2}\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$

Integration by Parts: $$\int_a^b u(x)\frac{\mathrm{d}v(x)}{\mathrm{d}x}\,\mathrm{d}x = u(x)v(x)\Big|_a^b - \int_a^b \frac{\mathrm{d}u(x)}{\mathrm{d}x}v(x)\,\mathrm{d}x$$

Series Expansions

Taylor series: $f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots$

Binomial expansion: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$ and $\binom{n}{k} = \frac{n!}{(n-k)!k!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots \qquad (|x| < 1)$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$, $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ and $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

$\ln(1+x) = \log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \qquad (|x| < 1)$

Geometric series: $\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1-r}$

Stirling's formula: $\log_e N! = N \log_e N - N$ or $\ln N! = N \ln N - N$

Trigonometry

$\sin(a \pm b) = \sin a \, \cos b \pm \cos a \, \sin b$

$\cos(a \pm b) = \cos a \, \cos b \mp \sin a \, \sin b$

$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \, \tan b}$

$\sin 2a = 2 \sin a \, \cos a$

$\cos 2a = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$

$\sin a + \sin b = 2 \sin \tfrac{1}{2}(a+b) \cos \tfrac{1}{2}(a-b)$

$\sin a - \sin b = 2 \cos \tfrac{1}{2}(a+b) \sin \tfrac{1}{2}(a-b)$

$\cos a + \cos b = 2 \cos \tfrac{1}{2}(a+b) \cos \tfrac{1}{2}(a-b)$

$\cos a - \cos b = -2 \sin \tfrac{1}{2}(a+b) \sin \tfrac{1}{2}(a-b)$

$e^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\, \sin\theta$

$\cos\theta = \frac{1}{2}\left(e^{\mathrm{i}\theta} + e^{-\mathrm{i}\theta}\right)$ and $\sin\theta = \frac{1}{2\mathrm{i}}\left(e^{\mathrm{i}\theta} - e^{-\mathrm{i}\theta}\right)$

$\cosh\theta = \frac{1}{2}\left(e^{\theta} + e^{-\theta}\right)$ and $\sinh\theta = \frac{1}{2}\left(e^{\theta} - e^{-\theta}\right)$

Spherical geometry: $\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$ and $\cos a = \cos b \, \cos c + \sin b \, \sin c \, \cos A$

Vector Calculus

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z = A_jB_j$$

$$\mathbf{A}\times\mathbf{B} = (A_yB_z - A_zB_y)\,\hat{\mathbf{i}} + (A_zB_x - A_xB_z)\,\hat{\mathbf{j}} + (A_xB_y - A_yB_x)\,\hat{\mathbf{k}} = \epsilon_{ijk}A_jB_k$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$$

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$$

$$\text{grad}\,\phi = \nabla\phi = \partial_j\phi = \frac{\partial\phi}{\partial x}\,\hat{\mathbf{i}} + \frac{\partial\phi}{\partial y}\,\hat{\mathbf{j}} + \frac{\partial\phi}{\partial z}\,\hat{\mathbf{k}}$$

$$\text{div}\,\mathbf{A} = \nabla\cdot\mathbf{A} = \partial_jA_j = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\text{curl}\,\mathbf{A} = \nabla\times\mathbf{A} = \epsilon_{ijk}\partial_jA_k = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\,\hat{\mathbf{i}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\,\hat{\mathbf{j}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\,\hat{\mathbf{k}}$$

$$\nabla\cdot\nabla\phi = \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$\nabla\times(\nabla\phi) = 0 \qquad \text{and} \qquad \nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$