PHY420 Problems Class 2: Biopolymers

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1. **Normalise 3D Gaussian distribution** The probability of an end to end vector $\boldsymbol{R}$ for a 3 dimensional random walk of $N \gg 1$ steps is given by the Gaussian distribution:

$$P(\boldsymbol{R}) \sim e^{-3\boldsymbol{R}^2/(2Na^2)}.$$

   Normalise this.

2. **Force-extension curve of a flexible polymer**

   (a) From the distribution given in question 1, find the entropy and from this the free energy of stretching a Gaussian polymer to $\boldsymbol{R}$, which is the end to end distance under the influence of the force f. From the free energy find the force f required to stretch the polymer.

   (b) The partition function for a flexible, freely jointed, chain subjected to a constant stretching force f, is given by:

$$Z = \int \mathrm{d}^2 a_i \int \mathrm{d}^2 a_2 \ldots \int \mathrm{d}^2 a_N e^{-\frac{E}{k_B T}}$$

   where

$$E = -\boldsymbol{f} \cdot \boldsymbol{R} = -\sum_{i=1}^{N} f a \cos\theta_i$$

   and $\int \mathrm{d}^2 a_i$ is the integral over volume in spherical coordinates for $r = a$. Keep f as f here (don't substitute in your answer to the previous part). Calculate this partition function and from it the extention given by:

$$\langle R \rangle = k_B T \frac{\partial}{\partial f} \ln Z.$$

   (c) Show that for small forces the answer in (b) gives the same as (a).