PHY221 Classical Physics
Tutorial questions

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Autumn semester

These questions cover all PHY221 tutorials (weeks 3, 8 and 11). The week each question should be able to be done is indicated at the beginning of the question.

Harmonic oscillators (week 3)

1. [Week 3] Imagine Doctor Who is in his TARDIS and has run out of fuel. He falls into a hole, which has been drilled by aliens, through the centre of the Earth from the North to the South pole. This problem will work out his fate. Assume the hole is frictionless and the Earth is a perfect sphere of radius R_E, mass M_E, and uniform density. Take the mass of the falling body (the TARDIS) to be $m_b \ll M_E$ and its position $r(t)$, the distance from the centre of Earth at time t. The modulus of the force of gravity, F_{grav}, acting on the TARDIS at any point $r(t)$, is given by $F_{\text{grav}} = -\frac{G m_b m_{in}}{r(t)^2}$, where G is the gravitational constant and m_{in} is the mass inside a sphere of radius $r(t)$ around the centre of the Earth.

   (a) Sketch the described situation, labelling all relevant physical quantities.

   (b) Calculate m_{in} as a function of $r(t)$.

   (c) Write Newton's equation of motion for the TARDIS and show that Doctor Who will oscillate forever (i.e. show that the equation of motion is an equation for a harmonic oscillator).

   (d) What is the frequency ω and period T of the oscillations?

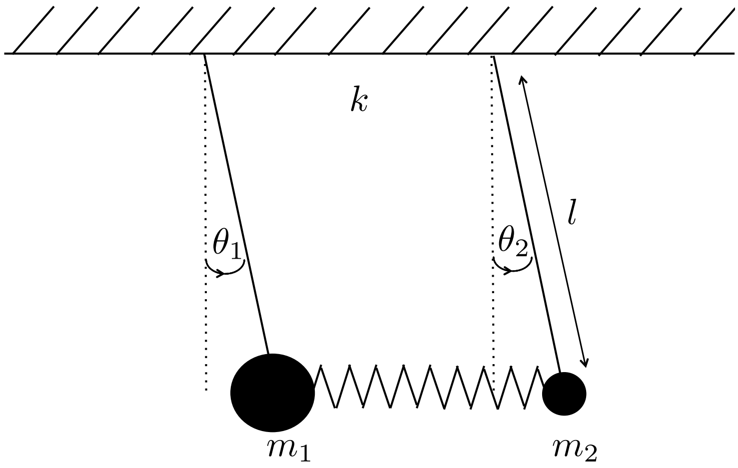
2. [Week 3] There is a fire door in your block of flats that bangs shut loudly and then bounces open again before closing properly. It wakes you up early every morning when your neighbour leaves for work.

   (a) Write the equation of motion for the door assuming it is a damped harmonic oscillator with mass $m = 10\,\mathrm{kg}$, spring constant $k = 3000\,\mathrm{N/\,m}$ and damping factor $\gamma = 10\,\mathrm{s}^{-1}$.
   (b) Show that the door is underdamped.
   (c) What should you adjust the damping factor to in order to make the door close as fast as possible without banging and bouncing open again?
   (d) Solve the equation of motion in this case of the door closing perfectly. Assume that at time $t = 0$ the door is wide open with an amplitude A_0 and is let go of (i.e. it's velocity is zero at time $t = 0$).
   (e) Sketch the graph of the door amplitude against time t.

3. [Week 3] A pneumatic drill is driven by a sinusoidal force of amplitude $F_{\max}$. The resonant frequency is 36 Hz. If a constant force $F_{\max}$ is applied the drill point is displaced by $3.0\,\mathrm{mm}$ from its equilibrium position. If the drill is driven with a frequency of 35 Hz or 37 Hz it oscillates with half the intensity it has at resonance. You may assume weak damping.

   (a) What will the amplitude be when driven at the resonance frequency 36 Hz?
   (b) Explain why we shake things that are stuck.

4. [Week 3] Consider 2 coupled pendulums, as shown in the diagram. Each pendulum has length l. One has a bob of mass m_1 and the other has a smaller bob of mass m_2. They are connected together by a spring of spring constant k. You may assume small angles and zero damping.



   (a) Write down the coupled equations of motion for the system.
   (b) Assume the solution has the form $x_1 = l\theta_1 = A_1 e^{i\omega t}$, $x_2 = l\theta_2 = A_2 e^{i\omega t}$ where A_1 is the amplitude of the first pendulum and A_2 that of the second. Solve the equations by putting them into matrix form to find the angular frequencies ω.
   (c) Find the amplitudes $A_{1,2}$ for each angular frequency ω.
   (d) Describe the resulting motion of the system in each case.

Waves (week 8)

5. [Week 5] Consider a flag blowing in the wind. Assume the transverse wave propagating along the flag is one dimensional.

   (a) Solve the wave equation for the wave on the flag, assuming the displacement of the flag is zero at the flag pole and the other end of the flag is free. The initial condition of the free end of the flag has maximum displacement, $y_{\max}$, and zero velocity.

   (b) What are the possible values of the wavelength of the wave, λ, in terms of the length of the flag?

6. [Week 5] There are two types of waves on water surfaces, depending on their wavelength compared to a characteristic wavelength, λ_{char}. For long waves ($\lambda \gg \lambda_{\text{char}}$), surface tension can be neglected. For short waves ($\lambda \ll \lambda_{\text{char}}$), which are also called 'capillary waves', surface tension makes an important contribution to the wave's energy. The two types of wave have different phase velocities, c_{long} and c_{cap}, given as a function of wavelength by the following equations:

$$c_{\text{long}} = \sqrt{\frac{g\lambda}{2\pi}}$$
$$c_{\text{cap}} = \sqrt{\frac{2\pi\sigma}{\rho\lambda}}$$

   where λ is the wavelength, $g = 9.81\,\text{m}\,\text{s}^{-2}$, $\rho = 1000\,\text{kg}\,\text{m}^{-3}$ is the density of water, and σ is a constant that quantifies the strength of surface tension and for water is $\sigma = 0.07\,\text{N}\,\text{m}^{-1}$.

   (a) Derive an expression for the group velocity of long water waves.

   (b) Derive an expression for the group velocity of capillary waves.

   (c) The characteristic wavelength λ_{char}, can be obtained by equating c_{long} to c_{cap}. Express λ_{char} in terms of σ, ρ and g and hence calculate the value of λ_{char} of water.

7. [Week 6] Imagine you are deep sea diving (and therefore under water) and your friend is in the rescue boat above the water. Your friend sees a shark coming and shouts to warn you, but you can't hear! Why not?

   (a) Calculate how much quieter the sound is for you. You will need to know the speed of sound in air, $c_{\mathrm{air}} = 330\,\mathrm{m/s}$, speed of sound in water, $c_{\mathrm{water}} = 1485\,\mathrm{m/s}$, density of air, $\rho_{\mathrm{air}} = 1.3\,\mathrm{kg/m^3}$ and density of water $\rho_{\mathrm{water}} = 1000\,\mathrm{kg/m^3}$.

   (b) Express your answer also in decibels.

Fictitious forces

8. [Week 9] You work for a pharmaceutical company and you need to separate particles from a suspension. The particles have mass $1\,\mathrm{ng}$. You put the test tube containing the suspension in a centrifuge with a rotational speed of $450\,\mathrm{rps}$ (rotations per second).

   (a) Calculate the centrifugal force on a $1.0\,\mathrm{ng}$ particle if the test tube is placed $20\,\mathrm{cm}$ away from the axis of rotation.

   (b) How does the magnitude of this force this compare to the weight of the particle?

   (c) You are now given a new sample with heavier particles of mass $0.010\,\mu\,\mathrm{g}$. If you place the new sample also at $20\,\mathrm{cm}$ away from the axis of rotation, what rpm rotation speed should you set the centrifuge to, so that the particles experience the same force as the particles in the first sample?

   (d) You wish to put both samples in the centrifuge at the same time. What distance from the axis of rotation should you place the second sample so that the particles in both samples experience the same centrifugal force?

9. [Week 9] Consider a cyclist riding a bike. Assume the cyclist and bicycle are a point mass, $m = 20\,\mathrm{kg}$ at a height $h = 0.4\,\mathrm{m}$ above the ground. The cyclist rides with velocity $v = 2.5\,\mathrm{m\,s^{-1}}$ in a circular path of radius $R = 3.0\,\mathrm{m}$ turning left (anticlockwise).

   (a) What is the magnitude and direction of the centrifugal force due to the curved path of the bike?

   (b) What is the torque experienced by the cyclist on the bike? Consider the torque about the point on the ground between the bike wheels. (Remember, if a force is applied at point B, the torque about point A is the cross product of the vector $\vec{AB}$ and the force.)

   (c) The cyclist naturally leans to the left to counteract the torque. At what leaning angle, θ (measured from vertical), will the cyclist experience no torque?

(d) If the cyclist puts on weight how will this affect the angle he/she should lean?

10. [Week 9] Consider a wheel rotating about a vertical axis with constant angular velocity ω. An ant is crawling radially outwards along one of the spokes of the wheel with a constant speed v'.

    (a) Find all the apparent forces acting on the ant, denoting the real force exerted on the ant by the spoke as $\boldsymbol{F}$ and consequently write Newton's equation of motion for the ant.

    (b) Given the coefficient of static friction μ between the ant and the spoke, how far can the ant crawl before it starts to slip?

11. [Week 9] The England captain claims it was the Coriolis force that lifted his penalty over the crossbar.

    (a) Can a Coriolis force lift a ball at all?

    (b) If so, does it depend on what location on Earth you are, and into what direction you shoot?

    (c) A penalty is taken $11\,\mathrm{m}$ from goal, and we assume the ball travels with $22\,\mathrm{m\,s^{-1}}$ along the line from where the penalty is take to the goal, plus an additional vertical velocity component that takes it in an almost straight line towards the crossbar. Estimate the maximum Coriolis lift, assuming the "worst case" location and shooting direction, and comment on the captain's excuse.

12. [Week 9] On its way to Paris the Eurostar train is travelling due South at $300\,\mathrm{km/h}$ at a point with latitude $49°$. Assume the Earth is a perfect sphere of radius, R_E, that rotates around its axis (North/South pole) once every 23 hrs 56 min.

(a) Calculate the magnitude of the acceleration of the train due to the Coriolis force. Note that latitude is defined as the angle ϕ between the normal to the surface of the earth and the equatorial plane such that $\phi = 0$ at the equator and $\phi = 90°$ at the North pole.

(b) What direction is this acceleration in?

Lagrangian (week 11)

13. [Week 11] A yo-yo is a toy consisting of a cylindrical disc of radius R, mass m, with moment of inertia, $I = \frac{1}{2}mR^2$. A massless string is wound around the rim of the cylindrical disc. The end of the string is fixed to the ceiling, and the disc of the yo-yo is allowed to fall under the influence of gravity. Use the Lagrangian method to work out the acceleration of the centre of mass of the yo-yo in terms of the acceleration due to gravity g.

14. [Week 11] Atwood's machine is a device with two weights (of mass m_1 and m_2) connected by an ideal (massless, inextensible) string of length l that passes over a frictionless pulley of radius a and moment of inertial I where m_p is the mass of the pulley.

    (a) How many degrees of freedom does the system have?

    (b) Construct the Lagrangian for the Atwood machine.

    (c) Using the Lagrange equations of motion, find the acceleration.

    (d) Discuss the behaviour of the system when $m_1 > m_2$, $m_1 < m_2$ and $m_1 = m_2$.

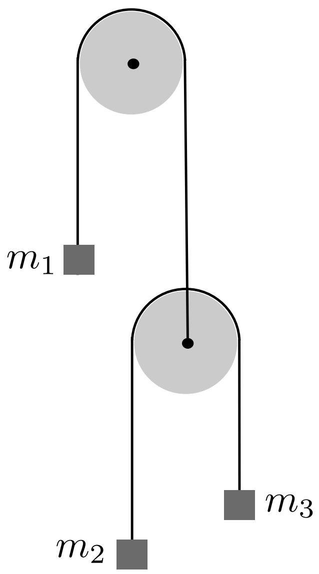
    (e) What effect does the moment of inertia of the pulley have?

    A double Atwood machine is made by replacing one of the weights by a second Atwood machine (as shown in the figure).

    (a) How many degrees of freedom are there now?

    (b) For simplification, in this case, assume the pulleys are massless and small ($a \ll l$) so you may neglect the moment of inertia and the bit of string around each pulley. Construct the Lagrangian for the double Atwood machine.

    (c) Using the Lagrange equations of motion, find the accelerations.


m_1
m_2
m_3