PHY221 Classical Physics **Problem Class 3**
Lagrangian mechanics & Fictitious Forces

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1. Consider a conical pendulum, which is a pendulum that traces a circle in the horizontal plane whilst being held at a fixed angle θ_0 from the vertical. The pendulum has a mass m and rod of length l.
   (a) What is the kinetic energy of the system?
   (b) What is the potential energy of the system?
   (c) Write down the Lagrangian.
   (d) Find the equation of motion from this Lagrangian.
   (e) Find a constant of motion and explain what its physical meaning.

2. Imagine a spaceship designed to simulate the effect of gravity during a mission to another plant (e.g. in the recent film "Interstellar"). The spaceship is built in the shape of a wheel with the astronauts living in a ring-shaped corridor in the "tyre".
   (a) i. How can a fictitious force be used to simulate gravity in a spaceship of this design?
   ii. What physical quantity determines the magnitude of the acceleration, a, due to this fictitious force for a spaceship of radius R?
   iii. Give the numerical values of this quantity required to simulate gravity of $a = g = 9.81\,\mathrm{m\,s^{-2}}$ if $R = 10\,\mathrm{m}$, $30\,\mathrm{m}$ and $100\,\mathrm{m}$.
   (b) i. What other fictitious force will astronauts experience when they walk along the corridor?
   ii. What is the magnitude and direction of this force for an astronaut walking at $1\,\mathrm{m\,s^{-1}}$ along the corridor in both directions? Use the simulated gravity and radii values given in (a).
   iii. Which value of R minimises this distortion of simulated gravity?
   (c) To keep fit the astronauts want to play table tennis.
   i. Show that even for the best choice of R, a table tennis ball flying down the corridor at a realistic $30\,\mathrm{ms^{-1}}$ will experience an acceleration stronger than simulated gravity.
   ii. Will this make on-board table tennis impossible? Discuss the possibilities and limitations.