PHY221 Classical Physics **Problem Class 2**
Coupled Oscillators & Waves

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1. Consider a system of coupled harmonic oscillators composed of 2 masses, m_1 and m_2 connected by a spring of spring constant k. In addition mass m_1 is connected to a wall by a spring, also of spring constant k.

   (a) What are the equations of motion of the system?

   (b) Solve these equations of motion to obtain the angular frequencies of the normal modes.

   (c) Show that, in the case in which $m_1 = m_2 = m$, these angular frequencies are given by $\omega^2 = \frac{k}{2m}(3 \pm \sqrt{5})$.

   (d) For this case of equal masses find the relationship between the amplitudes of the oscillations for each mode and consequently describe the modes.

2. Following an accident a patient has an ultrasound scan on their leg to check for muscle damage. The medical ultrasound test uses ultrasound of frequency $11.5\,\mathrm{MHz}$. The density of muscle is $1.12\,\mathrm{g\,cm^{-3}}$ and the density of the surrounding leg tissue is $0.940\,\mathrm{g\,cm^{-3}}$. Assume the ultrasound wavelength is $\lambda = 0.138\,\mathrm{mm}$ in muscle and $\lambda = 0.132\,\mathrm{mm}$ in the surrounding tissue. Calculate the intensity of the ultrasound image of the muscle compared to that of the incident intensity. You will need to use the formula for the reflection coefficient

$$r = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

   where Z_1 and Z_2 are the impedances of the soft tissue and the muscle.

3. Imagine listening to a conversation the other side of a solid metal door.

   (a) The dispersion relation for sound travelling in a metal is given by

$$\omega^2 = \frac{K}{2m}\left(1 - \sqrt{1 - \sin^2\frac{ka}{2}}\right)$$

   where ω is the angular frequency, k is the wavenumber and m is the mass of the metal atoms, which are separated by a distance a and connected by springs of spring constant K. Find the group velocity in this case.

   (b) Find an expression for ω in the case of small k.

   (c) Compare the group velocity and phase velocity of sound in this metal for the case of small k.

   (d) Explain what will happen to the low and high frequency components as the sound travels through the metal.

   (e) Why do the voices sound quieter to you listening behind the metal door? Give an expression for how much quieter the voices sound, defining the symbols you use.