PHY221 Classical Physics **Problem Class 1**
Harmonic Oscillators

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1. In the absence of damping a bungee jumper of mass, m on the end of an elastic cord of spring constant k, would continue to bounce up and down forever with simple harmonic oscillations with angular frequency, ω_0.

   (a) Given that the bungee jump cord has natural length l and is attached at height h above the ground write down the equation of motion for the position z above the ground of the jumping person.

   (b) By find the equilibrium point show that this equation is equivalent to the simplest simple harmonic oscillator equation in terms of ω_0.

   (c) Show that the total energy of such a simple harmonic oscillator is constant.

2. You are cycling home from the supermarket with 30kg of food and drink in carrier bags hanging on your handle bars and your housemate sat on the back of your bike. Assume your bike suspension is critically damped for a load of 90kg.

   (a) Show that your bike is now underdamped.

   (b) You ride over a pothole, displacing your position to $z_{\max}$ at time $t = 0$. Show that you now oscillate by solving the equation of motion. Express your answer in terms of $z_{\max}$ and express the damped angular frequency ω_d of the overloaded bike in terms of the undamped frequency of the bike without the extra load.

   (c) How much longer will the overloaded system take to return to the equilibrium position compared to the critically damped bike?

   (d) Sketch the graph of amplitude z against time t.

3. Consider a quartz clock. The oscillations (expansion and contraction) of the quartz crystal have spring constant k and damping factor γ. The oscillator is driven by an electrical driving force, $F_0 e^{i\omega t}$.

   (a) Solve the equation of motion for this driven oscillator to find the general solution for the transient decaying oscillations.

   (b) Solve the equation of motion for long times, $t \gg \frac{1}{\gamma}$, when the transient oscillations have decayed away.

   (c) Find the maximum amplitude of the oscillations and the angular frequency at which this occurs.

   (d) Given that the natural frequency of the quartz crystal is $f = 33\,\mathrm{kHz}$, and the damping factor $\gamma = 0.20\,\mathrm{s}^{-1}$, what is the quality factor, Q?