

PHY221 Classical Physics **Homework 3**

Fictitious forces and Lagrangian mechanics

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Deadline: 4pm Monday 14th December 2015

1. A child slides down the playground slide, which has a length of 4.2 m and is inclined at an angle of $\alpha = 25^\circ$ to the horizontal ground. You notice that the slide is not fixed but is slipping horizontally along the muddy ground as the child slides down. Assume the child (of mass $m = 15$ kg) slides with no friction and the slide (of mass $M = 200$ kg) slips in the mud with no friction.
 - (a) Sketch a diagram of the system, clearly labelling the generalised coordinates. [2]
 - (b) What is the child's kinetic energy and the total kinetic energy of the system? [4]
 - (c) What is the Lagrangian of the system? [2]
 - (d) Write down the equations of motion. [2]
 - (e) Solve the equations to find expressions for the child's acceleration and that of the slide. Use the numbers given above to find the numerical values of these accelerations. [6]
 - (f) Assuming the child and the slide start at rest, calculate the time it takes the child to slide from the top to the bottom of the slide and the distance the slide moves in this time. [4]

2. Consider a person riding a bike with velocity $v = 3.2 \text{ m s}^{-1}$ in a circular path of radius $R = 4.5$ m turning left (anticlockwise). Assume the person and the bike are a point mass, $m = 80$ kg at a height $h = 0.6$ m above the ground.
 - (a) What is the magnitude and direction of the centrifugal force due to the curved path of the bike? [4]
 - (b) What is the torque experienced by the cyclist? Consider the torque about the point on the ground between the bike wheels. (Remember, if a force is applied at point B, the torque about point A is the cross product of the vector \vec{AB} and the force.) [4]
 - (c) The cyclist leans to the left to counteract the torque. At what leaning angle, θ (measured from vertical), will the cyclist experience no torque? [10]
 - (d) If the cyclist puts on weight over Christmas how will this affect the leaning angle? [2]