PHY221 Classical Physics **Homework 1**
Oscillations

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Deadline: 4pm Monday 19th October 2015

1. Imagine you are responsible for a small child (of mass $15\,\mathrm{kg}$) in a children's playground. The child sits on a swing of length $2.5\,\mathrm{m}$. You pull the swing seat to a height of $10\,\mathrm{cm}$ above its resting position and let go. Assume weak damping with a friction force $F_f = -cv$ where v is the velocity and $c = 2.4\,\mathrm{kg\,s^{-1}}$.

   (a) Draw a labelled diagram of the system. Label the angle θ of the swing measured from vertical. [2]

   (b) Write down Newton's second law of motion for the system in terms of the angle θ. Explain the meaning of each term in the equation. [6]

   (c) Assuming the amplitude of the swing is small, show how the equation you wrote in part (b) can be simplified to the standard damped harmonic oscillator equation. [2]

   (d) Show that $\theta = Ae^{-\gamma t + i\omega_d t}$ is a solution to your equation in part (c). [4]

   (e) How long will it take for the amplitude of the oscillations to decrease to a value (10%) of its starting value? [2]

   (f) How many oscillations is this? [2]

   (g) With what frequency f should you push the swing so it oscillates with the maximum amplitude? [2]

2. A fire alarm consists of a small, electrically driven, hammer hitting a bell. The system has a spring constant k and damping factor γ. The oscillator is driven by an electrical driving force, $F_0 \cos(\omega t)$. The equation of motion for the system is:

$$\frac{\mathrm{d}^2\mathrm{x}}{\mathrm{dt}^2} + 2\gamma\frac{\mathrm{dx}}{\mathrm{dt}} + \frac{kx}{m} = \frac{F_0\cos(\omega t)}{m}$$

   (a) Find the general solution for the transient decaying oscillations. [4]

   (b) Solve the equation of motion for long times, $t \gg \frac{1}{\gamma}$, when the transient oscillations have decayed away. Hint: use the trial solution $x(t) = B_1\cos(\omega t) + B_2\sin(\omega t)$ and find the constants B_1 and B_2. [6]

   (c) Rewrite the solution to part (b) in the form $x(t) = A\cos(\omega t + \phi)$. [2]

   (d) Find the resonant angular frequency ω_r for which A in part (c) is maximum. [4]

   (e) The electrical driving frequency is $f = 50.0\,\mathrm{Hz}$, the mass is $m = 1.50\,\mathrm{g}$ and the damping factor is $\gamma = 20.0\,\mathrm{s^{-1}}$. What spring constant would give the best design? [4]