The University Of Sheffield.

Data Provided:
A formula sheet and table of physical constants is attached to this paper.

DEPARTMENT OF PHYSICS AND ASTRONOMY

Autumn (2013)

Classical Physics 2 hours

Instructions:
Answer question ONE (Compulsory) and 2 other questions.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

Please clearly indicate the question numbers on which you would like to be examined on the front cover of your answer book. Cross through any work that you do not wish to be examined.

COMPULSORY

1. You go for a day out in the countryside.

(a) You notice the leaves on a tree oscillate with a frequency of $4\,\mathrm{Hz}$. Assuming simple harmonic motion with damping factor $\gamma = 0.2\,\mathrm{s}^{-1}$, what is the quality factor, Q, for this system? **[1]**

(b) One leaf falls off the tree and spins (rotates) about itself as it falls vertically downwards. What is the Lagrangian for the falling leaf? Define the symbols you use. **[3]**

(c) It rains and the leaves on the tree get wet. You notice droplets of water move horizontally off the leaves when the branches swing along a one-dimensional horizontal curved path. What force causes the water droplets to move off the leaves and what is its origin? **[2]**

(d) Use dimensional analysis to find an expression for the Stokes drag force acting on a falling raindrop. **[2]**

(e) You hear thunder. Briefly explain the physics behind why you hear the sound continue for several seconds and not a short (or instantaneous) sound. **[2]**

SECTION A - answer 2 questions from this section

2. A cyclone is a type of storm that approximates to a horizontal circle of moving air. It is caused by low pressure air surrounded by higher pressure.

(a) Sketch a diagram of the cyclone clearly showing/describing the horizontal forces acting and the direction of air velocity. Assume the cyclone is in the Northern Hemisphere. **[3]**

(b) The wind (air) speed is $75\,\mathrm{km\,h^{-1}}$ and the pressure difference between the centre and the edge of the cyclone is $10\,\mathrm{Pa}$. The density of air is $\rho_{air} = 1.3\,\mathrm{kg\,m^{-3}}$. Estimate the diameter of the cyclone. **[3]**

(c) What are the forces acting in the vertical direction? State their directions. **[2]**

(d) Explain why the cyclone would rotate in the opposite direction if it was in the Southern Hemisphere. **[2]**

3. You see a rainbow.

(a) Derive the wave equation for the electric field strength E of light from Maxwell's equations in one dimension;

$$\frac{\partial B}{\partial t} = -\frac{\partial E}{\partial x} \tag{1}$$

$$\frac{\partial E}{\partial t} = -\frac{1}{\epsilon\mu}\frac{\partial B}{\partial x} \tag{2}$$

where B is the magnetic field strength, $\epsilon = \epsilon_r\epsilon_0$ is the dielectric permittivity and $\mu = \mu_r\mu_0$ is the magnetic permeability (subscripts r mean relative and subscripts 0 refer to that of free space). **[3]**

(b) What is the phase velocity, c, of light in a medium in terms of the phase velocity in free space, c_0 and the refractive index $n = \sqrt{\epsilon_r\mu_r}$ of the medium? **[1]**

(c) What is the group velocity? Express your answer in terms of n and the wavelength λ. **[3]**

(d) Calculate the refractive index of water, n_w, assuming $n_w = 1.365 - 52\,000\,\lambda$ for blue ($\lambda = 450\,\mathrm{nm}$) and red ($\lambda = 650\,\mathrm{nm}$) light. **[1]**

(e) Calculate the refraction angles of red and blue light in water for an incident angle of $60.0°$. **[1]**

(f) What fraction of blue light is transmitted through a water droplet (i.e. from air to water to air)? Note the transmission coefficient $t = \frac{4Z_1Z_2}{(Z_1+Z_2)^2}$ where $Z = \sqrt{\frac{\mu_r\mu_0}{\epsilon_r\epsilon_0}} = c_0\mu_0/n$ is the optical impedance of the medium. **[1]**

4. Consider a carbon dioxide molecule assuming it has a linear structure (a carbon atom bonded to an oxygen atom either side of it such that the two bonds lie on the same axis). Treat each bond as a spring of spring constant k.

(a) Write down the coupled differential equations for the longitudinal vibrations of this system (i.e. oscillations parallel to the axis of the bonds). **[2]**

(b) Solve these equations to find the angular frequencies of the longitudinal modes of vibration. **[4]**

(c) Using your answer to part (b) find the numerical values of the vibrational frequencies, f, in Hz. Assume the spring constant of each bond is $k = 46\,\mathrm{N\,m^{-1}}$, the mass of an oxygen atom is $16\,\mathrm{u}$ and the mass of a carbon atom is $12\,\mathrm{u}$, where u is given in the formula sheet. **[2]**

(d) Using your answers, explain why carbon dioxide is a "greenhouse gas" contributing to global warming (climate change). Compare this to nitrogen, $\mathrm{N_2}$. **[2]**

5. Consider the rotation of the Earth and its motion around the Sun.

(a) What is the Lagrangian for the Earth? You may assume that the translational motion of the Earth is in a plane and the mass of the Earth is much less than that of the Sun so the Sun is fixed at the centre of the system (i.e. a central force field). **[2]**

(b) From this Lagrangian derive the equations of motion for the system. **[3]**

(c) State the mathematical form and physical meaning of the constants of motion. **[3]**

(d) Given that the angle between the angular velocity of the Earth about its axis and the angular velocity of the Earth about the Sun is 23.5°, explain why the number of hours of daylight varies at different times of year in Sheffield. **[2]**

END OF EXAMINATION PAPER