The University Of Sheffield.

Data Provided:
A formula sheet and table of physical constants is attached to this paper.

DEPARTMENT OF PHYSICS AND ASTRONOMY

Autumn 2012

Topics in Classical Physics — 2 hours

Instructions:
Answer question 1 (compulsory) and 2 other questions.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

COMPULSORY

1. (a) When watching the diving at the Olympics the crowds cheer when the diver finishes the dive, but at this point the diver is underwater.

i. Use dimensional analysis to derive an expression for the speed of sound in water in terms of the density ρ and the bulk modulus $K = -V\frac{\mathrm{d}P}{\mathrm{d}V}$ where V is the volume and P is the pressure. **[2]**

ii. Find out how much quieter the cheering sounds to the diver underwater by calculating the transmission coefficient,

$$t = \frac{4Z_{air}Z_{water}}{(Z_{air} + Z_{water})^2},$$

of sound wave intensity from air to water, assuming normal incidence ($Z_{air/water}$ is the impedance of air/water). The speed of sound in water is $c_{water} = 1500\,\mathrm{m\,s^{-1}}$, the speed of sound in air is $c_{air} = 330\,\mathrm{m\,s^{-1}}$, density of water is $\rho_{water} = 1000\,\mathrm{kg/\,m^{-3}}$ and the density of air is $\rho_{air} = 1.2\,\mathrm{kg/\,m^{-3}}$. **[2]**

(b) i. In the discus throw events the athlete spins around with the discus (a circular disc of mass m) in their hand such that the discus traces out a horizontal circle. A simplified model of this system can be given by the Lagrangian:

$$L = \frac{1}{2}ml^2\dot{\theta}^2\sin^2\alpha_0 - mgl(1 - \cos\alpha_0)$$

where l is the length of the athlete's arm, θ is the angular coordinate around the horizontal circle and α_0 is the angle the athlete's arm makes with the vertical. Find the constant of motion and state its physical meaning. **[2]**

ii. The athlete throws the discus in such a way that as it travels through the air it spins clockwise with angular frequency ω. Consider a droplet of water on the top of the discus, at a radius $\boldsymbol{r}$ from the centre of the discus. What forces are acting on the water droplet? Please give the magnitude and direction and define all symbols used. **[3]**

(c) A gymnast swings from a bar as if they were a simple pendulum. If the gymnast has a mass $m = 50\,\mathrm{kg}$ and height $1.5\,\mathrm{m}$ what is the period of their oscillations? **[1]**

SECTION A - answer 2 questions from this section

2. Track cycling is held on a track that is steeply banked at an angle of 45°. In this question assume a cyclist on their bike is a point mass $m = 70\,\text{kg}$ at a height $h = 0.50\,\text{m}$ vertically from the ground and that the track is frictionless. The curved sections of the track have a radius of $R = 20\,\text{m}$ (measured to the point mass of the cyclist).

(a) Draw a diagram of the system clearly labelling the forces acting on the cyclist. **[2]**

(b) Find an expression for the torque acting on the cyclist. Remember if a force $\boldsymbol{F}$ is applied at point B about a pivot point A the torque is defined as $\boldsymbol{T} = \boldsymbol{r} \times \boldsymbol{F}$ where $\boldsymbol{r}$ is the vector from A to B. **[2]**

(c) Use your previous answer to find the velocity at which the bike will be stable when perpendicular to the track. Express your answer in km/h. **[2]**

(d) A top racing speed may be 85 km/h. At this speed how much will the cyclist have to lean and in what direction? **[2]**

(e) As the cyclist slows down to stop what happens to the leaning angle? Explain your answer. **[2]**

3. In springboard diving (illustrated in the figure) the diver bounces on the end of the springboard in order to gain extra height to perform the dive.

(a) Treating the springboard as a damped harmonic oscillator driven sinusoidally by the diver, write the equation of motion, explaining what each term means. **[2]**

(b) By substituting in the trial solution $A(\omega)e^{i(\omega t - \phi)}$ find the amplitude and phase of the springboard as a function of the driving angular frequency ω. **[4]**

(c) With what period should the diver periodically force the board to make the board oscillate with the maximum possible amplitude? Assume the spring constant is $3000\,\text{N}\,\text{m}^{-1}$, the effective mass is 70 kg and its damping factor is $0.10\,\text{s}^{-1}$. **[2]**

(d) What is the quality factor of this springboard? **[1]**

(e) The spring constant can be adjusted between $2000 - 4000\,\text{N}\,\text{m}^{-1}$. Why would a diver want to change the spring constant? **[1]**

4. When an expert diver lands in the water they cause only a small splash. This produces ripples which spread out across the pool. The wave equation for the height h of such ripples (capillary waves) is given by

$$\frac{\partial^2 h}{\partial t^2} = \left(\frac{\sigma k}{\rho_{water} + \rho_{air}}\right) \nabla^2 h$$

where k is the wavenumber, $\sigma = 0.072\,\mathrm{N/m}$ the surface tension, $\rho_{water} = 1000\,\mathrm{kg\,m^{-3}}$ the density of water and $\rho_{air} = 1.2\,\mathrm{kg\,m^{-3}}$ the density of air.

(a) What is the phase velocity? **[1]**

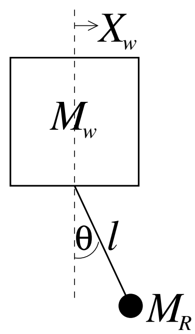
(b) What is the dispersion relation? **[1]**

(c) What is the group velocity? How does this compare to the phase velocity? **[2]**

(d) How long will ripples of wavelength $\lambda = 1\,\mathrm{cm}$ take to reach the edge of the pool 5 m away? **[2]**

(e) For the 1D case $\nabla^2 = \frac{\partial^2}{\partial x^2}$. Solve the wave equation assuming the splash at time $t = 0$ results in the height h being an even function in space x. Leave your answer as a function of k and arbitrary coefficients A_k. Explain why we would not expect the answer to be a pure cosine wave. **[4]**

5. A Paralympic athlete has a point mass $M_R = 500\,\mathrm{g}$ hanging from their wheelchair from an inextensible massless string of length $l = 20\,\mathrm{cm}$. The wheelchair with the athlete sitting in it has a mass $M_W = 80\,\mathrm{kg}$. The mass swings in the vertical plane without being forced. Assume this system can be modelled as a simple pendulum hanging from a movable support.



(a) Show that the Lagrangian for this system is:

$$L = \frac{1}{2}(M_W + M_R)\dot{X}_W^2 + \frac{1}{2}M_R(l^2\dot{\theta}^2 + 2\dot{X}_W\, l\, \dot{\theta}\cos\theta) + M_R\, g\, l\cos\theta$$

where X_W is the horizontal distance the wheelchair moves and θ is the angle the pendulum makes with the vertical. **[4]**

(b) Work out what the equations of motion for this system are. **[2]**

(c) Assuming a small angle approximation and that the nonlinear term is negligible, solve for θ and find the angular frequency in $\mathrm{rad\,s^{-1}}$. **[2]**

(d) Find X_W and hence the amplitude of distance oscillations of the wheelchair given that the maximum amplitude of the pendulum is 10°. **[2]**

END OF EXAMINATION PAPER