PHY221 Classical Physics **Essential equations**

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Harmonic oscillators

$$\text{Newon's law} \quad F = ma = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$$
$$\text{Hooke's law} \quad F = -kx$$
$$\omega = \frac{2\pi}{T} = 2\pi f$$

- Simple harmonic oscillator

$$\text{equation:} \quad \frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \omega_0^2 x = 0$$
$$\text{mass-spring:} \quad \omega_0 = \sqrt{\frac{k}{m}}$$
$$\text{pendulum:} \quad \omega_0 = \sqrt{\frac{g}{l}}$$
$$\text{solution:} \quad x(t) = x_{\max}\cos(\omega_0 t + \phi)$$

- Damped harmonic oscillator

$$\text{equation:} \quad \frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\gamma\frac{\mathrm{d}x}{\mathrm{d}t} + \omega_0^2 x = 0$$
$$\gamma = \frac{c}{2m}$$
$$\text{solution for weak damping:} \quad x(t) = x_{\max}e^{-\gamma t}\cos(\omega_d t + \phi)$$
$$\omega_d = \sqrt{\omega_0^2 - \gamma^2}$$
$$\text{critial damping when } \gamma^2 = \omega_0^2$$

- Driven harmonic oscillator

$$\text{equation:} \quad \frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\gamma\frac{\mathrm{d}x}{\mathrm{d}t} + \omega_0^2 x = \frac{F_0}{m}e^{i\omega t}$$
$$\text{solution:} \quad x(t) = A(\omega)e^{i(\omega t - \phi(\omega))}$$

Maximum when driving freqency ω equals the resonance frequency:

$$\omega_r = \sqrt{\omega_d^2 - \gamma^2}$$

- Quality factor

$$Q = \frac{A(\omega_r)}{A(0)} \approx \frac{\omega_0}{2\gamma}$$

$$Q = \pi N \quad \left(N \text{ oscillations for amplitude to decay to } \frac{1}{e}\right)$$

$$Q = \frac{\omega_r}{\Delta\omega_{\text{FWHM}}}$$

Waves

$$\text{wave equation: } \quad \frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}$$

$$\text{solution: } f(\phi) \text{ where } \phi = kx \pm \omega t$$

$$c = \lambda f \qquad k = \frac{2\pi}{\lambda} \qquad \omega = ck$$

$$\text{dispersion relation } \omega(k) = c(k)k$$

$$\text{phase velocity } c = \frac{\omega}{k}$$

$$\text{group velocity } v_G = \frac{\mathrm{d}\omega}{\mathrm{d}k}$$

$$\text{impedance } Z = c\rho$$

$$\text{reflection/transmission } r + t = 1$$

Fictitious forces

$$\text{Centrifugal force } \boldsymbol{F'}_{\text{cf}} = -m\,\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r'})$$

$$\text{Coriolis force } \boldsymbol{F'}_{\text{cor}} = -2m\,\boldsymbol{\omega} \times \boldsymbol{v'}$$

Lagrangian mechanics

$$L = T - V$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}$$

If $\frac{\partial L}{\partial q_i} = 0$ then q_i is cyclic/ignorable and $\frac{\partial L}{\partial \dot{q}_i}$ is a constant of motion.

$$\text{Translational kinetic energy } T = \frac{1}{2}mv^2$$

$$\text{Rotational kinetic energy } T = \frac{1}{2}I\omega^2$$

$$\text{Gravitational potential energy } V = -\frac{GM_1M_2}{r}$$